Math 15
Sections 1 & 2
Fall 2009

Name _____

Exam II

INSTRUCTIONS: This exam is seven pages long; check that you have all the pages. Be sure to show all your work on the attached sheets. No credit will be given for unsubstantiated solutions. Also, where appropriate, please indicate your final answers by circling them.

You may use a calculator, but **for arithmetic only**—no graphing, equation solving, or calculus-based features. Mark the places where you've used a calculator by writing a "C" in the left margin. Feel free to ask during the exam if any questions arise about calculator use (or about anything else for that matter).

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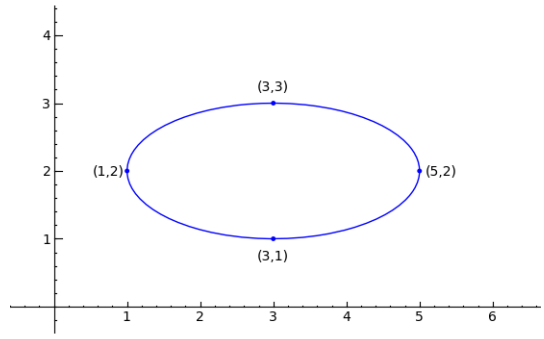
1. Evaluate: $\lim_{x \to 1} \dfrac{x^3 - 3x + 2}{x^3 - 3x^2 + 2}$. (8 points)

1.	
2.	
3.	
4.	
5.	
6.	
7.	
Total	**/90**

2. **(a)** Write a parametrization for the ellipse shown below, traced out counterclockwise. As usual, use the letter t as parameter. (16 points)



(b) For your parametrization in part (a), what point on the ellipse corresponds to $t = 2$? What is the slope of the curve at that point?

3. A motorist traveling at a speed of 40 ft/sec sees a moose crossing the road ahead and applies the brakes so as to decelerate at a constant rate of 20 ft/sec^2. How far has the car advanced by the time it has slowed to 10 ft/sec? (12 points)

4. While on a road trip, a Prius owner notices that, as the car consumes gasoline, its fuel efficiency in miles per gallon (mpg) decreases slightly, as shown in the following table.

(16 points)

Gasoline consumed (gallons)	Fuel efficiency (mpg)
0	48
2	46
4	45
6	43
8	41
10	40

Let $E(x)$ denote the fuel efficiency in mpg when the car has consumed x gallons of gasoline.

(a) Estimate the integral $\int_0^{10} E(x)\,dx$ as accurately as the data allows. Be sure to include enough work so that it is clear how you got your answer.

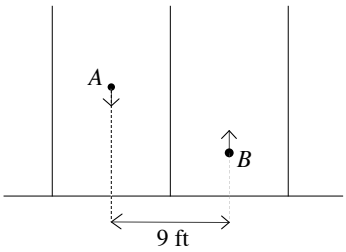
(b) Explain in words what the integral $\int_0^{10} E(x)\,dx$ represents. Be sure that your reasoning is clear.

5. A cup of coffee cools as it sits on a table. The coffee's temperature $T(t)$ changes at a rate of $T'(t) = -65\,e^{-t}$ °C/minute, where t is measured in minutes. The coffee's temperature is 90°C when $t = 0$. (16 points)

(a) Find a formula for the coffee's temperature at time t.

(b) What is the coffee's average temperature from $t = 0$ to $t = 3$? Justify your answer fully by showing all steps.

6. Two swimmers are racing in adjacent lanes. Swimmer A approaches one end of the pool, while Swimmer B has already reached that end and has turned around and is swimming the other way. The swimmers swim right down the middle of their respective lanes, which are 9 feet wide. At a certain instant, swimmer A is 6 feet from the wall and swimming at a rate of 7 ft/sec, while swimmer B is 2 feet from the wall and swimming at 5 ft/sec. How fast is the distance between the swimmers changing at that moment? Is the distance increasing or decreasing? (*Hint:* Find an appropriate right triangle.) (12 points)



7. **(a)** State the fundamental theorem of calculus. (10 points)

(b) Explain *why* the fundamental theorem is true. For instance, you could discuss how it follows from Euler's method for constructing an anti-derivative. Your solution will be graded for correctness, completeness, and logical clarity. This almost surely involves writing in complete sentences. By the same token, you will lose points for false or sloppy statements.